Towards a geometric understanding of the space of stochastic processes

Joint Supervision by Julio Backhoff and Mathias Beiglböck (UV)

Stochastic Processes are a ubiquitous tool in mathematics and its applications. Despite decades of research, a deep understanding of the *geometry of the space of stochastic processes* is still pending. One reason is that, until very recently, it was common for researchers to consider stochastic processes as nothing more than a random variable taking values in a set of possible paths. Despite its appealing simplicity, this reduction *from stochastic processes to random variables* dismisses inherently important structures in stochastic analysis such as filtrations, the arrow of time, stopping times, etc. The nascent field which studies stochastic processes, without stripping away their probabilistic properties, is called *causal optimal transport* (COT). The expectation is that this dissertation will help expand this field.

Initially the aim of the dissertation is to establish important elementary results for COT, comparable to those known for the now classical field of *Optimal Transport* (OT). For instance, we would like to answer: When do we need external sources of randomization in order to optimally couple two stochastic processes? Is the optimality of a coupling a property of its *weighs* or of its *support*? Once this is settled, the aim is to advance the understanding of the geometry of the space of stochastic processes equipped with the natural metric that is built from COT: the *adapted Wasserstein distance*. For instance: how does the tangent space to a stochastic process look like? Can we characterize the geodesics between two stochastic processes? When are these unique? What is the right notion of barycenter for a family of stochastic processes?

The above is a suggestion which can be adapted to the successful candidate's interests and strengths.