Fast Continuous Time Dynamics for Monotone Inclusions
(Supervisor: R.I. Boţ)

Inertial gradient systems with asymptotically vanishing viscous damping are known to accelerate the convergence of the values of the underlying convex function along the trajectories to its minimum. Time discretizations of these systems provide fast converging algorithms with momentum terms which have been widely used in the last decade to solve optimization problems arising in engineering and machine learning. Recently, it has been surprisingly discovered that inertial dynamical systems with vanishing damping term governed by a single-valued (possibly non-potential) monotone operator exhibit fast convergence rates for the norm of the operator, whereas the trajectories converge to one of its zeros ([2], see also [3]). The associated implicit and explicit numerical algorithms share the asymptotic features of the continuous dynamics, which proves that inertial/momentum effects act as convergence accelerators beyond the setting of smooth convex optimization.

The main topic of this PhD thesis project is the study of fast continuous dynamics associated to monotone inclusions governed by set-valued maximally monotone operators. Monotone inclusions emerge, for instance, as systems of optimality conditions of nonsmooth convex composite optimization problems, of regularized saddle point problems, and of constrained convex optimization problems can be represented, provided that regularity conditions are satisfied, and this makes their investigation of particular interest. To the questions to be addressed belong the asymptotic behavior of the trajectories and the impact of Tikhonov regularization on it, the derivation of asymptotic rates of convergence for fixed point residuals and corresponding gap functions, the connection with first order dynamics via open loop and/or closed loop ([1]) time rescaling and averaging techniques. Special emphasis will be given to the derivation by temporal implicit and/or explicit discretization of fast numerical algorithms and the study of their convergence properties.

The PhD candidate is expected to have a solid theoretical background in optimization theory, functional analysis and numerical analysis as well as strong programming skills.

[1] H. Attouch, R.I. Boţ, E.R. Csetnek (2022): *Fast optimization via inertial dynamics with closed-loop damping*, Journal of the European Mathematical Society, DOI: 10.4171/JEMS/1231

[2] R.I. Boţ, E.R. Csetnek, D.-K. Nguyen (2022): *Fast OGDA in continuous and discrete time*, arXiv:2203.10947

[3] R.I. Boţ, D.-K. Nguyen (2022): *Fast Krasnosel'skii-Mann algorithm with a convergence rate of the fixed point iteration of o(1/k)*, arXiv:2206.9462