Basic and elliptic Appell hypergeometric series

PhD Project supervised by
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The four Appell series F_1, F_2, F_3 and F_4 are double-series extensions of the Gauß hypergeometric ${}_2F_1$ series. They were introduced by Paul Émile Appell in 1880 who showed that they are the analytic solutions of certain partial differential equations that extend the hypergeometric equation (satisfied by the Gauß hypergeometric series). Many formulas for the Appell hypergeometric series exist which connect these bivariate series to the classical univariate theory. To single out a specific useful result, there exists, in particular, a remarkable expansion formula for an F_4 series in terms of multiples of products of two ${}_2F_1$ series, due to Burchnall and Chaundy. A special case of this expansion formula can be used to compute the generating function for Jacobi polynomials (which form an important family of orthogonal polynomials).

It is well known that in a single variable the theory of hypergeometric series naturally extends to the more general theories of *basic* (or *q*-)hypergeometric series and further to *elliptic* hypergeometric series. While basic hypergeometric series have been systematically studied since the 1840s (starting with Heine, Gauß, etc.), elliptic hypergeometric series (EHS) are a relatively new subject. They first appeared explicitly in 1997 in work of I. Frenkel and V. Turaev (who identified EHS as elliptic solutions of the Yang–Baxter equation and were the first to give explicit transformation and summation formulae satisfied by them).

It is natural to ask about basic and elliptic extensions of Appell hypergeometric series. Whilst results for basic Appell hypergeometric series do exist, not many are known. The few existing summations and transformations naturally have the flavour of being sporadic. Carrying out investigations on a systematic scale should lead to the discovery of more results and a better understanding of the basic theory, which is one of the main goals of the PhD project.

Now, for elliptic Appell hypergeometric series essentially nothing is known, although some (non-Appell type) elliptic double sum identities have been found. The other main goal of this project is to carry out systematic investigations towards the emergence of a theory of elliptic Appell hypergeometric series (and corresponding integrals).

The potential PhD student should have a strong interest in special functions, and not be aversed to manipulating explicit expressions containing several variables. The PhD student would first gain a profound knowledge on the state of art of hypergeometric series, Appell series, basic and elliptic hypergeometric series, and what is known about basic Appell series. This would serve as a basis for making the sought advances in the theory of basic Appell hypergeometric series and for developing a theory of elliptic Appell hypergeometric series.