Spherical Design Curves for Continuous Measurements in Spatial Hearing

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Efficient sampling on the sphere is a key challenge in many measurement applications. Recent work on spherical t-design curves shows that continuous trajectories can provide optimal sampling guarantees while drastically reducing acquisition time. This project transfers these ideas to acoustics and head-related transfer function (HRTF) measurements, which constitute the basis of spatial hearing and are typically performed at finitely many static directions. Here, those static samples are replaced by mathematically designed mobile sampling schemes.

Project Aims

Construct and analyze spherical t-design curves for continuous HRTF sampling, and validate them in a moving-loudspeaker setup.

- *Construct spherical t-design curves and hybrid sampling schemes:* Spherical t-design curves provide optimal sampling guarantees for low-bandwidth spherical data [1]. Key questions concern the construction of short, practically realizable curves, including weighted and hybrid designs [2, 3] that combine continuous trajectories with a small number of static sampling points.

- *Spherical harmonic models and curve-based sampling:* HRTFs are often represented via spherical harmonic expansions, effectively yielding bandlimited functions on the sphere. How can the sampling guarantees of t-design curves be adapted to such bandlimited HRTF models? Which assumptions on the HRTF field [4] are required to obtain stable reconstruction from samples taken along a single closed curve?

- *Simple proof-of-concept measurement setup:* As a first engineering step, we consider a far-field loudspeaker mounted on a rotating arc around a listener, following a smooth trajectory of a spherical t-design curve. We record continuous responses and reconstruct direction-dependent HRTFs by mapping time to direction [5], thereby testing the theoretical predictions.

Candidates should have a strong mathematical background, ideally with prior exposure to Fourier or harmonic analysis, approximation theory, numerical analysis, or sampling theory. Programming skills (e.g. Python or Julia) are desirable but not mandatory. A strong interest in interdisciplinary work at the interface of mathematics, signal processing, and acoustics is expected, as the project is embedded in the research environment of ARI and AHA, with close supervision by both advisors.

References

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[4] P. Majdak, P. Balazs, and B. Laback. Multiple exponential sweep method for fast measurement of head-related transfer functions. *Journal of the Audio Engineering Society*, 55(7/8):623–637, 2007.

[5] M. Pollow. Fast measurement of individual head-related transfer functions. *PhD thesis*, RWTH Aachen University, 2019.