Project title: Combinatorial Sets of Reals
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This project focuses on the so-called combinatorial sets of reals, sets of reals that usually originate in analysis, topology, and algebra (see [5]) and are often associated with the combinatorial cardinal characteristics of the continuum. Their study brings into focus the infinitary combinatorial properties of the real line, properties that often manifest a common combinatorial structure among seemingly unrelated mathematical objects. A good example is provided by maximal cofinitary groups and meager subsets of the real line. Recall that a cofinitary group is a subgroup of S_∞, the group of permutations of the natural numbers, with the property that every non-identity element of the group has only finitely many fixed points. A maximal cofinitary group is a cofinitary group that is maximal under inclusion. The minimal cardinality of a maximal cofinitary group is denoted $\mathfrak{a}_g$. The collection of meager subsets of the reals is denoted $\mathcal{M}$, and the minimal cardinality of a non-meager set is denoted $\mathrm{non}(\mathcal{M})$. A celebrated theorem of Brendle, Spinas, and Zhang states that $\mathfrak{a}_g \leq \mathrm{non}(\mathcal{M})$ (see [2]). That is, a cofinitary group of cardinality smaller than $\mathfrak{a}_g$ is not only necessarily non-maximal but also meager. An excellent exposition of the combinatorial cardinal characteristics of the continuum can be found in [1], while further discussion of combinatorial sets of reals and their origins can be found in [5].

The axiomatic system of Zermelo–Fraenkel set theory, together with the method of forcing (for background, see [4, 3]), captures a rich landscape of infinitary combinatorial structures associated with the sets of reals described above. The goal of this project is to advance our understanding of these structures in the following ways: **(A)** investigating the consistency of constellations among the combinatorial cardinal characteristics of the continuum, including constellations in which three or more characteristics take distinct values; **(B)** analyzing the spectra of these characteristics, that is, the possible cardinalities of the associated combinatorial sets; **(C)** examining the projective complexity of the corresponding sets of reals; **(D)** studying analogues of the classical combinatorial sets of reals in the higher Baire spaces, which are not only of independent interest but often reveal striking distinctions between the infinitary combinatorics at ω and at uncountable cardinals κ.

The method of forcing is a powerful combinatorial tool that not only allows one to establish relative consistency results but, through its combinatorial nature, also serves as a generalization of inductive constructions. The project builds to a great extent on the interaction of three large and distinct areas of set theory - set theory of the reals, descriptive set theory, and large cardinals - thereby creating opportunities and the foundation for broad and fruitful collaborations.

[1] A. Blass *Combinatorial cardinal characteristics*, Handbook of set theory, 395–489, Springer, 2010.

[2] J. Brendle, O. Spinas, Y. Zhang *Uniformity of the meager ideal and maximal cofinitary groups*, J. Algebra, 232 (1):209–208, 2011.

[3] T. Jech *Set theory*, Springer Monographs in Mathematics, Springer, 2003.

[4] K. Kunen *Set theory*, Studies in Logic (London), 34, College Publications, 2011.

[5] J. Steprans *History of the continuum in the 20th century*, Elsevier, 2012.