Modern aspects of non-smooth Lorentzian geometry

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This PhD project will explore aspects of non-smooth Lorentzian geometry, the mathematical theory underlying Einstein's general relativity. Just as metric length spaces provide a synthetic generalisation of smooth Riemannian manifolds, the time-separation function plays the role of a "distance" in Lorentzian geometry. The need for a non-smooth Lorentzian theory appeared early on, most famously through Penrose's singularity theorems, and remains central to understanding low-regularity spacetimes. At least two research directions are available, sub-Lorentzian geometry and causal set theory. Other topics in the area of non-smooth Lorentzian geometry may also be chosen, depending on the student's interests. The project will be carried out in collaboration with Dr. Samuël Borza.

Sub-Lorentzian geometry studies causal geometry on manifolds where motion is constrained to a non-integrable distribution (only certain directions are allowed), and the time-separation is measured by a Lorentzian metric defined on those allowed directions. Although it is the Lorentzian analogue of sub-Riemannian geometry, spacetimes with non-holonomic constraints have received comparatively little attention. The project will investigate curvature notions in constrained spacetimes, isoperimetric-type problems, and how sub-Lorentzian geometry can provide a unified geometric framework for general relativity coupled to Maxwell's theory of electromagnetism.

Causal set theory is a radical approach to quantum gravity in which spacetime is modelled as a discrete causal graph. A new notion of curvature, inspired by Ollivier-Ricci curvature and based on optimal transport between causal diamonds, has recently been developed here at the University of Vienna, and this project will investigate key open questions such as a timelike Bonnet-Myers theorem and the discrete-to-continuum limit. A further aim is to connect these ideas with central problems in the field. For instance, the Benincasa-Dowker conjecture predicts that the expectation of the causal set action, averaged over random "sprinklings" into a globally hyperbolic Lorentzian spacetime, converges in the continuum limit to the usual Einstein-Hilbert action of that spacetime.

Expected Background

Applicants are expected to have a strong mathematical background and a solid command of at least one, and preferably several, of the following areas: Riemannian or semi-Riemannian geometry, metric geometry, optimal transport, and the mathematical foundations of general relativity.