Logic and Dynamics

Advisor: Aristotelis Panagiotopoulos

This call is for students who are interested in working at the interface between mathematical logic and dynamics. The relation between the two is reciprocal. In one direction, mathematical logic provides powerful tools for analyzing dynamics of large topological groups. These tools include **Fraïssé theory**, **continuous logic**, **Baire-category methods**, and **Banach space theory** and are particularly indispensable in the absence of local compactness, where the lack of a Haar measure renders classical methods of harmonic analysis and measure theory inadequate. In the other direction, the "wild" dynamics of topological group often constitute the primary reason as to why some "classification problems" from mathematics tend to be high up in the *Borel reduction hierarchy*: a hierarchy whose study forms one of the most prominent ongoing research programs of **descriptive set theory**.

Several currently very active research threads have emerged from this cross-pollination, providing applications —and uncovering further connections— across a range of areas in mathematics and mathematical physics, including **topology** (algebraic topology, manifold topology, and theory of continua), **representation theory**, and **general relativity**. To name just a few representative directions:

1. **General Relativity.** Descriptive set theory was recently used to give a negative solution to the longstanding open *Problem of Observables* in general relativity [PSC23]. Ongoing work further reveals connections between the theory of *Lorentzian length spaces* and *continuous logic*.
2. **Definable Algebraic Topology.** It was recently shown that several classical invariants from homological algebra and algebraic topology can be enriched with (Borel) definable content, yielding much stronger means of classification [BLP24a, BLP24b].
3. **Representation theory.** The Gelfand–Raikov theorem has recently been extended beyond the realm of locally compact groups [Tsa12]. At the same time, new examples of *exotic* Polish groups have been discovered, i.e. groups admitting no non-trivial unitary representations.
4. **Continua and manifolds.** Projective Fraïssé theory has emerged as a useful tool for studying homeomorphisms of continua [PS22], while the Borel reduction hierarchy has been recently used to establish anti-classification results in manifold theory (e.g. smooth structures on $\mathbb{R}^4$)
5. **Pure set theory.** New connections between descriptive set theory and cardinal characteristics of the continuum have recently emerged [PS23], while the theory of generalized Baire spaces provides a natural setting for extending results from the descriptive set theory of Polish groups.

We invite applications from PhD students interested in working in logic and dynamics. The precise thesis topic will be developed jointly with the successful applicant, drawing on the many open problems and follow-up directions arising from **any of the research threads outlined above**.

Co-supervision. Joint supervision is very welcome and explicitly encouraged.

References

[BLP24a] J. Bergfalk, M. Lupini, and A. Panagiotopoulos, *The definable content of homological invariants I: Ext and* $\lim^1$, Proceedings of the London Mathematical Society **129** (2024), no. 3, e12631.

[BLP24b] J. Bergfalk, M. Lupini, and A. Panagiotopoulos, *The definable content of homological invariants II: Čech cohomology and homotopy classification*, Forum of Mathematics, Pi **12** (2024).

[PS22] A. Panagiotopoulos and S. Solecki, *A combinatorial model for the Menger curve*, Journal of Topology and Analysis **14** (2022), no. 1, 203–229, DOI 10.1142/S1793525320500478.

[PSC23] A. Panagiotopoulos, G. Sparling, and M. Christodoulou, *Incompleteness Theorems for Observables in General Relativity*, Physical Review Letters **131** (2023), 171402.

[Tsa12] T. Tsankov, *Unitary representations of oligomorphic groups*, Geometric and Functional Analysis **22** (2012), no. 2, 528–555, DOI 10.1007/s00039-012-0156-9.

[PS23] Márk Poór and Saharon Shelah, *On the weak Borel chromatic number and cardinal invariants of the continuum* (2023), available at `2302.10141`. arXiv:2302.10141.