Extendibility of spacetimes and Lorentzian length spaces

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The theory of Lorentzian length spaces[1] has recently been introduced as a framework that generalizes both Lorentzian geometry beyond the smooth manifold setting and metric geometry to Lorentzian signature. Just as metric geometry has led to identifying the 'metric core' of many results of Riemannian geometry, in particular through the synthetic approach to curvature bounds in the theory of Alexandrov and CAT(k)-spaces, as well as Ricci curvature bounds via optimal transport theory[3] in the framework of metric measure spaces, the theory of Lorentzian length spaces aims to provide similar tools for Lorentzian geometry. Despite being a recent development, the theory has already found several applications to relevant models of General Relativity and Lorentzian geometry[3].

The problem of spacetime extendibility[2] is central to General Relativity, particularly in connection with singularities and the cosmic censorship hypothesis. Understanding the conditions that allow or obstruct spacetime extension is essential to the physical interpretation of solutions to Einstein's equations. The aim of this thesis project is to develop and apply curvature comparison methods in Lorentzian length spaces to investigate the extendibility of spacetimes beyond smooth settings. A central focus will be to understand how synthetic curvature bounds constrain possible extensions of spacetimes, both in the classical manifold setting and in the more general framework of Lorentzian length spaces.

The basic requirement for the position is an excellent working knowledge in at least one, ideally several, of the following areas: (semi-)Riemannian geometry, metric geometry, optimal transport, and (mathematical) general relativity.

References

[1] M. Kunzinger and C. Sämann, Lorentzian length spaces, *Ann. Global Anal. Geom.* 54(3), 399-447 (2018).

[2] J.D.E. Grant, M. Kunzinger, and C. Sämann, Inextendibility of spacetimes and Lorentzian length spaces, *Ann. Global Anal. Geom.* 55, 133-147 (2019).

[3] F. Cavalletti and A. Mondino, Optimal transport in Lorentzian synthetic spaces, synthetic timelike Ricci curvature lower bounds and applications, *Cambridge J. Math.* 12(2), 417-534 (2024).