Elliptic combinatorics and special functions

PhD Project supervised by
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The aim of this project is to study certain combinatorial aspects of elliptic hypergeometric series and related objects, with the overall goal to further develop the rather new theory of elliptic hypergeometric series.

Elliptic hypergeometric series (EHS) are series where the quotient of two consecutive terms is an elliptic (i.e., doubly-periodic meromorphic) function in the summation index, the latter being considered as a complex variable. These special functions form a natural generalization of hypergeometric and basic hypergeometric series.

EHS appeared for the first time implicitly in 1987 in the work of the Japanese statistical physicists Date, Jimbo, Kuniba, Miwa and Okado while working on the Yang–Baxter equation. Ten years later, Igor Frenkel and Vladimir Turaev discovered explicit transformation and summation formulae satisfied by EHS. These results, which involve series satisfying modular invariance, are deep and elegant and help to better understand various phenomena of the simpler basic case. Moreover, the theory of EHS beautifully combines the theories of theta functions with the theory of basic hypergeometric series.

Over the last years, a substantial amount of the existing theory for hypergeometric and basic hypergeometric series has already been extended to the elliptic setting. However, many questions still remain open.

The aim of this project is two-fold:

1. One main goal is to study various connections of EHS to combinatorics. For instance, combinatorial enumeration using elliptic weights can be used to obtain elliptic extensions of special combinatorial numbers. Other objects of interest include elliptic commuting variables and the combinatorial study of theta function identities.

2. The other main goal is to study multiseries extensions of EHS. In particular, it would be worthwile to find elliptic extensions of the four Appell series F_1, F_2, F_3 and F_4 (which, in the classical case, can be viewed as double-series extensions of the Gauß hypergeometric ${}_2F_1$ series) and corresponding integrals, and establish various essential properties these series and integrals satisfy.

To achieve the specific objectives, a variety of methods will be used, ranging from combinatorial machinery to tools from algebra and classical analysis.

The potential PhD student should have a strong interest in hypergeometric series and their various extensions, and their connections to enumerative and algebraic combinatorics. The PhD student would first gain a solid knowledge on the state of art of hypergeometric, basic and elliptic hypergeometric series, and their diverse appearances in combinatorics. This would serve as a basis for making the sought advances in the combinatorial theory of EHS and for further developing the theory of EHS and its extensions to multiseries.