A gentle introduction to the world of colored trisps: from combinatorial topology to quantum gravity

Colored triangulated spaces (hereafter colored trisps) are a particular type of abstract simplicial complexes, that is, piecewise-linear spaces obtained by gluing simplices together. They were first introduced under the name *crystallizations* by the "Italian school" initiated by Pezzana and Gagliardi in the 1970's [Pez74; Gag79], with the goal of classifying (piecewise-linear) manifolds of low dimension. They later garnered interest from theoretical physicists, starting with Gurău [Gur11] in the context of colored tensor models, a recent approach to quantum gravity. Compared to other types of discrete spaces, the appeal of colored trisps stems from their natural combinatorial encoding by so-called colored graphs, which grants them many nice properties.

The goal of this course is to introduce the main concepts and results related to colored trisps, and to give a sense of the challenges arising in the study of discrete structures in dimension higher than 2.

We will start by introducing a few general notions related to simplicial complexes, before focusing on colored trisps. We will then express their main properties in terms of the associated colored graphs. Then, to showcase the power of using colored trisps to study discrete geometry, we will present results obtained on the number of (general) triangulations of the d-dimensional sphere thanks to colored trisps [Riv13; CP21]. We will then proceed to the study of random colored trisps: we will first give some physical motivations, that are related to quantum gravity, and introduce the necessary tools to study such questions. To put things into context, we will present known results in dimensions 1 and 2. Finally, we will present the results that have been obtained so far for scaling limits of random colored trisps [GR14; Car19], and mention some open problems.

Tentative schedule:
Lecture 1: General notions for simplicial complexes and a first look at colored trisps
Lecture 2: Important properties of colored trisps and colored graphs
Lecture 3: Using colored trisps to study general PL manifolds
Lecture 4: Studying scaling limits of random discrete stuctures: why and how
Lecture 5: Scaling limits of colored trisps

References

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