Approximation of Functions from Scattered Data

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A typical problem in science is the development of a theoretical model for a hidden process from observational data. More precisely, we are given a data set of measurements $\mathcal{D} = \{(x_j, f_j) : j = 1, \ldots, M\}$, where we assume that the sampling nodes $\mathcal{C} = \{x_j : j = 1 \ldots, M\}$ are a finite subset of a metric space $(\mathbb{X}, d)$ and the sampling values $\{f_1, \ldots, f_M\}$ are complex numbers.

Assuming that a hidden process generated the data means we suppose that there exists a function f which generated the observed data. This assumption obviously leads to the equations $f(x_j) = f_j,\ j = 1, \ldots, M$ or, even more realistic, $f(x_j) \approx f_j$ since mostly the data are corrupted with noise or measurement errors. Now our aim is to determine an approximation p to the function f using the given information $\mathcal{D}$ only. The approximation p will then be considered as a model for the underlying process f. Dealing with real world problems we can hardly expect that the sampling nodes are equally spaced or lying on a particular grid. Indeed, due to experimental constrains the sampling nodes are frequently scattered points. As one standard example one may think of measurement on the surface of the earth, where due to topographic conditions one is not able to collect data on every preferable point. Consequently, we have to deal with interpolation respectively approximation of functions from scattered data. We will discuss the various mathematical aspects of these type of approximation problems in the course.