



Vienna School
of Mathematics

PhD Colloquium

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The proof-theoretic strength of a theory.

Abstract: This talk concerns two measures of the strength of axiomatic theories based on the study of their proofs: the Π_1^1 -norm and the Π_2^1 -norm of a theory.

After giving a brief historical introduction to proof theory, we will see how Gentzen used the technique of cut-elimination to prove the consistency of Peano Arithmetic using transfinite induction up to the ordinal ε_0 . We will explain how this result led to the development of ordinal analysis and the notion of *proof-theoretic ordinal*, or Π_1^1 -norm, of a theory. As a generalization of this measure, we will present the Π_2^1 -norm, defined in terms of an object called *dilator*, which is a particularly uniform transformation of linear orders that preserves well-foundedness. A linear order is well-founded if any non-empty subset has a least element.

We will also introduce some subsystems of second-order arithmetic. These are theories that have two sorts of variables: first-order variables such as x, y, n, m to denote natural numbers and second-order variables X, Y, Z to denote sets of natural numbers. Many notions and theorems from mathematics can be formalized in this framework. We will present some of their Π_1^1 - and Π_2^1 -norms.

October 19, 2026, 16:00-17:00

TUForMath Room DAEGH18, Freihaus, TU Wien
(Wiedner Hauptstrasse 8-10)