REPULSIVE POINT PROCESSES IN FUNCTIONAL AND SIGNAL ANALYSIS

Proposal for a PhD project
Advisor: José Luis Romero, University of Vienna
https://sites.google.com/site/jlromeroresearch/

Random point processes are ubiquitous is statistical mechanics and have more recently attracted significant attention in data science. The purpose of this project is to study certain random configurations of points and their applications in analysis and signal processing. In contrast to a Poisson process, where the statistics of disjoint observation regions are independent, we will be interested in *repulsive processes*, where disjoint observation regions are negatively correlated. In the presence of repulsion, points are less likely to cluster and typical realizations are evenly spread.

The project can develop in many directions according to the candidate's interests, *with emphasis on either pure or applied mathematics.* We plan for a suitable co-advisor, possibly among the authors of [1, 2, 3, 4]. The following are two possible research lines.

Sampling, interpolation, and discrepancy. Given a class of functions and their values on a distinguished set (samples), the main questions are: Is every function determined by its samples? Can a function with prescribed samples be found? Configurations that solve such problems need to be rather uniformly distributed, and therefore repulsive processes are expected to provide high quality solutions. Conversely, we aim to explore fine properties of important repulsive point processes, such as discrepancy, by showing that they solve sampling and interpolation problems for appropriate function classes.

The Coulomb gas is a case in point [1]. The gas consists of a large number of repelling point charges confined by an external potential. At very low temperatures a certain rigid behavior (freezing regime) is expected to emerge. Statistical estimates supporting such intuition are derived in [1, 2], by showing that at low temperatures, a random sample of the Coulomb gas solves certain sampling and interpolation problems with weighted polynomials.

Zeros of time-frequency representations. While signals of interest are often oscillating, they tend to have a well-defined time-frequency profile. Recent research has brought to the foreground the rich information encoded in the zeros of time-frequency representations, because these exhibit repulsion in the presence of even a moderate amount of noise.

The exploitation of such insights is challenging, as it requires a computational link between analog domain statistics and finitely given data [3]. The PhD candidate is expected to contribute to the analysis of algorithms relevant in signal processing. The zeros of time-frequency representations have rich statistics, because they may have positive or negative winding numbers, and they exhibit different levels of repulsion accordingly, much in analogy to charged particles [4]. The exploration and exploitation of this phenomenon is another possible research direction.

Background and interest in some of the following areas is helpful: real, complex, harmonic and functional analysis, probability, statistics, information theory, and signal processing.

References

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[3] L. A. Escudero, N. Feldheim, G. Koliander, and J. L. Romero. Efficient computation of the zeros of the Bargmann transform under additive white noise. *Foundations of Computational Mathematics.* To appear. https://arxiv.org/pdf/2108.12921.pdf

[4] A. Haimi, G. Koliander, and J. L. Romero. Zeros of Gaussian Weyl-Heisenberg functions and hyperuniformity of charge. *Journal of Statistical Physics.* 187, 22 (2022). https://arxiv.org/pdf/2012.12298.pdf