Synthetic singularity theorems in Lorentzian length spaces

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The singularity theorems of General Relativity are one of the milestones of 20th century mathematical physics and establish that geodesic incompleteness of Lorentzian manifolds under physically reasonable conditions is generic. These results have recently been extended to non-smooth spacetimes by our research group, using distributional energy conditions/(Ricci) curvature bounds and analytic regularisation techniques.

On the other hand a theory of Lorentzian length spaces has recently been introduced also by our research group. It provides a framework analogous to metric length spaces, a well-established branch of geometry, which allows a synthetic approach to (sectional) curvature bounds in the theory of Alexandrov and CAT(k)-spaces. It has led to identifying the 'metric core' of many results in differential geometry, and to generalizations of central notions in the field to low regularity situations. Similarly, Lorentzian length spaces generalize Lorentzian geometry and causality theory beyond the setting of differentiable manifolds and spacetime-metrics.

To introduce Ricci curvature bounds in the synthetic setting Cavaletti and Mondino have transferred methods of optimal transport theory to Lorentzian length spaces. Indeed they have managed to adapt the core elements of the theory of CD and RCD spaces, which allows to characterize Ricci bounds in term of the convexity of an entropy functional. As an application they have also proved a first version of Hawking's singularity theorem in their framework.

The aim of this thesis is to explore the recent optimal transport based approach to synthetic singularity theorems due to Cavalletti and Mondino and to develop an overarching theory comprising also the distributional approach to this topic developed earlier by our research group.

Literature:

F. Cavalletti, A. Mondino, A review of Lorentzian synthetic theory of timelike Ricci curvature bounds. J Gen. Relativity Gravitation, 54(11), 137 (2022).

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