

## STABILITY OF WORD MAPS, WITH APPLICATIONS

**Context:** This project is in the area of Ulam’s stability of representations and a general “almost implies near” phenomenon. This is renowned thanks to a connection to physics, originally noticed by von Neumann in his approach to quantum mechanics (1929). This can be also viewed in a wider context of stability conceived by Ulam (1960): an equation is stable if an almost solution (or a solution of the corresponding inequality) is near an exact solution. An elementary example of a result of interest is as follows: almost commuting matrices are near matrices which commute. The word map in this example is the commutator word on two variables:  $XYX^{-1}Y^{-1}$ .

There is a precise relation to a recently emerged area of metric approximations of discrete groups. For instance, this concerns concepts of soficity (as studied by Gromov, Weiss, etc.) and hyperlinearity (in the setting of Connes’s embedding conjecture) which can be viewed as the *existence* (in contrast to *stability*) of such an “almost” object (in the above elementary example: the existence of a metric approximation of group elements by almost commuting matrices), see [1,3] and references therein.

**Major open problem:** Is there a non residually finite group which has stable groups relators (in certain family of matrices, with respect to a given norm, independently of matrix sizes)?

**Expected background of applicants:** students who like groups, matrices, combinatorics, low-dimensional geometry and who are motivated to learn new things in various exciting areas.

[1] G. Arzhantseva, L. Paunescu, *Linear sofic groups and algebras*, Transactions of the American Mathematical Society, 369 (2017), 2285–2310  
[doi:10.1090/tran/6706](https://doi.org/10.1090/tran/6706)

[2] G. Arzhantseva, L. Paunescu, *Almost commuting permutations are near commuting permutations*, Journal of Functional Analysis, 269(3) (2015), 745–757  
[doi:10.1016/j.jfa.2015.02.013](https://doi.org/10.1016/j.jfa.2015.02.013)

[3] G. Arzhantseva, *Asymptotic approximations of finitely generated groups*, Research Perspectives CRM Barcelona-Fall 2012 (Trends in Mathematics), Birkhäuser, Basel, vol. 1, 2014, 5–12  
[doi:10.1007/978-3-319-05488-9\\_2](https://doi.org/10.1007/978-3-319-05488-9_2)