Stochastic Gradient Descent Algorithms for Imaging Problems

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During the last decades several efficient imaging algorithms have been developed: Opposed to previous algorithms novel techniques contain stochastic features, which allow for computational cost efficient implementation. For instance in recent years the *ensemble Kalman inversion* (EKI) has attracted considerable attention for solving inverse problems [7], while previously it was used for dynamical filtering of time-series. EKI has advantages against other iterative methods in situations where the evaluation of the forward operator is costly, and information about its adjoint or its derivative are unavailable.

In [10] it has been shown that ensemble Kalman inversion (EKI) for **linear** inverse problems can equivalently be formulated as a stochastic low-rank approximation of Tikhonov regularization. This point of view allows us to improve practical performance. Furthermore, discrete adaptive versions of EKI, where the sample size is coupled with the regularization parameter, allow for proving for an order optimal regularization method under standard assumptions.

Concerning **nonlinear problems**, EKI has been studied as an optimization method [12, 13, 3]. However, so far, there are are only few results concerning regularization theory of EKI in the case of general convex regularization and more general for nonlinear inverse problems.

The starting point for this proposal are variants of EKI for solving convex imaging problems. We can rely on the identities between Tikhonov regularization in EKI and a further relation to Gauss-Newton methods as already pointed out in [10]. When general regularization terms are introduced in Tikhonov-regularization they are often solved with forward-backward splitting algorithms (see for instance [5]). One can therefore ask for generalizing EKI utilizing the coherence to Tikhonov-regularization, which can be solved for instance with stochastic forward backward gradient descent algorithms for monotone inclusion equations (see [1, 9, 2, 11, 4]).

In the long run, we hope that this analysis can provide us with the possibility of the development and analysis of stochastic algorithms for solving nonlinear inverse problems, like ultrasound tomography. We expect that the conditions in [6, 8] provide the key ingredients for an analysis.

References

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