A gentle introduction to Geometric Measure Theory

Fabian Rupp

Given a smooth closed curve $\Gamma \subset \mathbb{R}^3$, *Plateau's problem* asks for the surface of least area among all surfaces bounded by Γ. In the study of this problem, it turns out to be necessary to describe geometric properties of nonsmooth sets which, when relying on measure theoretical concepts, leads to the field of *Geometric Measure Theory.* The techniques developed in formulating and solving Plateau's problem have inspired various mathematical advancements and find applications also in Calculus of Variations, Partial Differential Equations, Optimal Transport, and Dynamical Systems, among others.

The goal of this course is to give an overview on the topic, to introduce the central objects of study, and to illustrate their powerful properties. We will start with *Hausdorff measures,* a way to measure the 'lower-dimensional volume' of sets in $\mathbb{R}^n$. Then, we turn to a class of sets that allow for a notion of tangent space termed *rectifiable sets,* and a weak form of the change of variables formula, the *(co-)area formula.* This allows us to examine the first variation of area, leading to a weak notion of curvature. We will then discuss compactness and regularity results for two crucial concepts of generalized submanifolds, *varifolds* and *currents*, and their role in solving Plateau's problem.

Tentative schedule:

Lecture 1 Hausdorff measures and rectifiability

Lecture 2 Area formula and first variation

Lecture 3 Varifolds: monotonicity formula and regularity

Lecture 4 Differential forms and currents

Lecture 5 Compactness and the Plateau problem