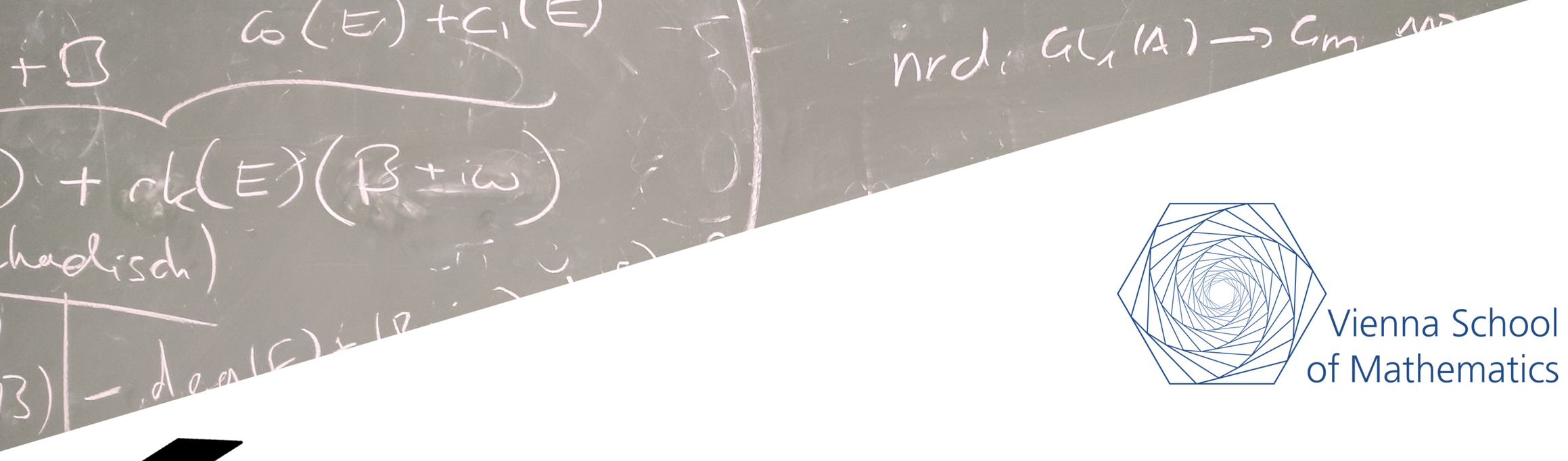

Vienna School of Mathematics

PhD Colloquium

Annamaria Massimini

The Life and Lies of Cross-Diffusion systems

Multispecies systems, i.e. systems of many species of interacting entities (for instance, particles, agents, cells or individuals), appear very often in sociology, biology and physiology, as well as in engineering contexts. A system of partial differential equations aims to describe the time evolution of the entities' concentrations on a macroscopic scale. On the one hand, species usually diffuse against their concentration gradient, as Adolf Fick stated in 1855. On the other, it could happen that the gradient of the density of one species induces a flux of another species. In these situations, cross diffusion occurs, leading to the so-called *cross-diffusion systems*, which are strongly non-linear and are composed of coupled partial differential equations. Cross-diffusion systems are challenging to study, since there are no standard techniques that can prove their well-posedness. Indeed, the diffusion matrix involves nonvanishing off-diagonal elements and, in many applications, it turns out to be neither symmetric nor positive definite, which considerably complicates the mathematical analysis. Moreover, due to the strong nonlinearity of the system, the maximum principle is not applicable to prove bounds for the species' concentrations.
In this talk, I will try to convince you that, if we also ask physics for help (alas), these systems are not so scary. It turns out that there is a (mathematical) *entropy* associated with such systems. As if we were using a Riddikulus spell, entropy allows us to introduce a change of variables that makes the diffusion matrix associated with the system semi-positive, and thus mathematically tractable! One can thus obtain, as if by magic, on the one hand, appropriate a priori estimates and, on the other, the possibility of studying the existence and uniqueness of a solution to the system.

23 March
15:00 – 15:45
Seminarraum AE U1 - 7
TU Wien